Test 2 Review

Dr. Graham-Squire, Spring 2012

•The test will cover sections 5.6-5.10, 6.1-6.4, and 6.6 (springs and ropes).

•To study, you should look over your notes, labs, rework HW problems, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapters 5 and 6 will also be good practice (For Chapter 5: page 424 #11-15, 17-23, pages 425-426 #22-35, 47(b), 48(b), 49(b), 55-60, 61 (for number 61, do a comparison by removing the +2 from the bottom), For Chapter 6: pages 488-489 #1-18, 20, 22, 23, 25, 27, 28)

•Calculators and/or Sage/Maple will be useful on this test, but for certain questions you will not be allowed to use them.

•Some practice problems to work on:

1. Use integration by parts to evaluate $\int (\ln x)^2 dx$.

   Ans: do integration by parts with $u = (\ln x)^2$ and $dv = dx$. Have to do integration by parts a second time as well, final answer is $x(\ln x)^2 - 2x \ln x + 2x + C$

2. Evaluate the integrals. Be sure to use correct notation where necessary.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{x^5}{\sqrt{4-x^2}}\, dx$ (Round to nearest .001)

   Ans: Use trig substitution (easier) or integration by parts (harder). Final answer is 0.758.

   (b) $\displaystyle\int \frac{x+21}{7x-x^2}\, dx$

   Ans: Use the method of partial fractions. final answer is $3\ln|x| + 4\ln|7-x| + C$.

   (c) $\displaystyle\int_0^2 \frac{3}{\sqrt{2-x}}\, dx$

   Ans: This is an improper fraction because the function is not continuous at 2. Need to use limit notation to get full credit, final answer is $6\sqrt{2}$

3. Use the Midpoint Rule with six subintervals (M_6) to approximate $\displaystyle\int_0^3 \frac{dt}{1+t^2+t^4}$. Then use Sage, Maple or a graphing calculator to calculate the actual integral.

   Ans: 0.89548 for M_6, actual numerical integral is 0.89537.

4. Sketch the region enclosed by $y = x^3 - 9x$ and $y = -5x$ and then find its area. (Note: the answer is not zero).

   Ans: Need to calculate $\displaystyle\int_{-2}^0 [(x^3-9x)-(-5x)]dx + \int_0^2 [(-5x)-(x^3-9x)]dx$. Final answer is 8.

5. Consider the region W bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$ and $x = 3$. Find the volume of the solid obtained by rotating W about (a) the line $y = -3$ and (b) the y-axis.

   Ans: (a) Using washers, get $\displaystyle\int_1^3 \pi[(3 + \frac{1}{x})^2 - 3^2]\, dx$.

   (b) Using shells, get $\displaystyle\int_1^3 2\pi x(\frac{1}{x})\, dx = 4\pi$.

6. Calculate the arc length of the curve $y = 4(x-3)^{3/2}$ for $3 \le x \le \frac{37}{12}$.

   Ans: Can take the integral by hand. final answer is 7/54.

7. A spring has a natural length of 50 cm. A 40-N force is required to stretch (and hold the spring) to a length of 75 cm. How much work is done in stretching the spring from 50 cm to 62 cm? Note: Since the force is given in Newtons, we need to express all lengths in meters.

   Ans: To find k, set $40 = k(0.25)$ to get $k = 160$. Then calculate $\displaystyle\int_0^{.12} 160x\, dx = 1.152$.

8. We have a cable that weighs 3 lbs/ft attached to a bucket filled with coal that weighs 700 lbs. The bucket is initially at the bottom of a 600 ft mine shaft. Answer each of the following.

   (a) Determine the amount of work required to lift the bucket to the midpoint of the shaft.

   Ans: Work for top half of rope + Work for bottom half of rope + work for bucket of coal = $\displaystyle\int_0^{300} 3x\, dx + 300(3)(300) + 700(300) = 615000$

   (b) Determine the amount of work required to lift the bucket from the midpoint of the shaft to the top of the shaft

   Ans: $\displaystyle\int_0^{300} 3x\, dx + 700(300) = 345000$

   (c) Determine the amount of work required to lift the bucket all the way up the shaft.

   Ans: $\displaystyle\int_0^{600} 3x\, dx + 700(600) = 960000$.